Limited-angle Multi-energy CT using Joint Clustering Prior and Sparsity Regularization

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Outline

- Introduction
- Methodology
- Experiments
- Summary

- **Spectral (Multi-energy CT)**
  - ▷ differentiates materials
  - ▷ wide applications: extracting veins and kedney stones, detecting chemical elements (iodine, barium)
- **Problems**
  - ▷ The data required for reconstruction is multiplied
  - ▷ longer scan time, more cost, more dose
- **Goal**
  - ▷ Design an easy-to-implement scanning strategy
  - ▷ Lower does, cost and acquisition/reconstruction time (less angular views)
  - ▷ mitigate limited-angle artifacts

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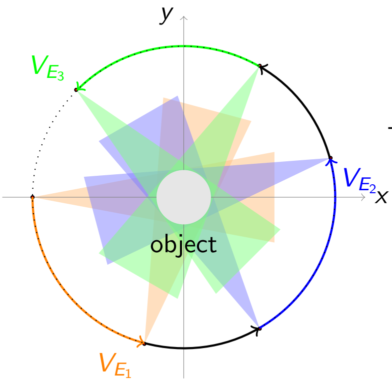
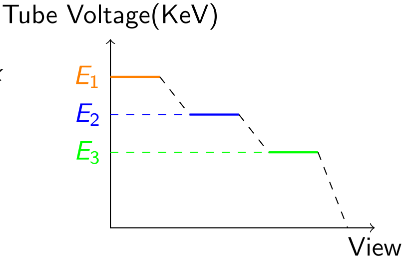


(a) Scanning trajectory



(b) Voltage switching

Requirement: The angular coverage of all X-ray beams is no less than 180° plus fan beam angle.

- Limited-angle problem:
  - ▷ violates data sufficient condition: 180° plus fan beam angle coverage
  - ▷ severe artifacts
  - ▷ hard to eliminate the artifacts using compressed sensing
- Solutions:
  - ▷ Independent reconstruction will unavoidablely encounter limited-angle artifacts.
  - ▷ Leverage the structural coherence of images at all energies
  - ▷ jointly reconstruct images at all energy channels

Combine the projection data from all energies to pre-reconstruct a prior image.

$$\boldsymbol{\mu}_p = \arg\min_{\boldsymbol{\mu}} \|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2$$

where

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{N_E} \end{pmatrix} \qquad \mathbf{p} = \begin{pmatrix} \mathbf{p}_1 / \|\mathbf{p}_1\|_1 \\ \mathbf{p}_2 / \|\mathbf{p}_2\|_1 \\ \vdots \\ \mathbf{p}_{N_E} / \|\mathbf{p}_{N_E}\|_1 \end{pmatrix}$$

Each indpendent $\mathbf{H}_i$ is ill posed, but the combined $\mathbf{H}$ not.

Figure : Prior image

L. Shen and Y. Xing, Medical physics, 42(1): 282, 2015

Solution:

$$\boxed{\text{Data fidelity} + \text{Regularization}}$$

Solution:

$$\boxed{\text{Data fidelity} + \overbrace{\text{Regularization}}^{\text{Prior} + \text{Sparsity}}}$$

- Assumption:
  1. The number of tissues within the object is limited;
  2. Each tissue is spatially continuous;
  3. The pixels within one tissues share an identical value.
- Clustering:
  1. $k-$means clustering on the prior image;
  2. choosing features $(x, y, \mu(x, y))$. (x, y): coordinates, $\mu(x, y)$: pixel values;
  3. The image is divided into k patches;
  4. These k patches keep some structural details.

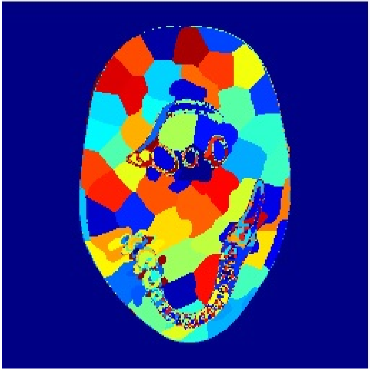


Figure : Clustering

Construct a constraint from the clustering

- k patches: $\Omega_c = \{(x,y) \mid \text{labeled with } c\}, c = 1,2,\ldots,k$.
- construct a dictionary

$$\boldsymbol{\mu} = \boldsymbol{\Phi}\mathbf{a} = \sum_{c=1}^{k} a_c \boldsymbol{\varphi}_c \tag{1}$$

where
$\boldsymbol{\Phi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_k) \in \mathcal{R}^{N\times k}$ dictionary matrix
$\boldsymbol{\varphi}_i$ basis vector (element)
$\varphi_{ij} = \mathbf{I}_{i\in\Omega_j} = \begin{cases} 1 & i \in \Omega_j \\ 0 & i \notin \Omega_j \end{cases}$

Joint Clustering Prior and Sparsity Regularization (CPSR) model

$$\arg\min_{\boldsymbol{\mu}} \frac{1}{2}\|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2 + \lambda\|W\boldsymbol{\mu}\|_1 \quad s.t. \quad \boldsymbol{\mu} = \mathbf{\Phi}\mathbf{a} \tag{2}$$

Incorporates the structural constraint into general compressed sensing frame.

- $\|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2$ linear projection model
- $\|W\boldsymbol{\mu}\|_1$ sparse constraint. W denotes wavelet transform.
- $\boldsymbol{\mu} = \mathbf{\Phi}\mathbf{a}$ structural constraint

Augmented Lagrangian Function:

$$L(\boldsymbol{\mu}, \mathbf{a}, \mathbf{z}, \mathbf{y}_1, \mathbf{y}_2) = \underbrace{\frac{1}{2}\|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2}_{\text{fedility}} + \underbrace{\frac{\rho_1}{2}\|\boldsymbol{\mu} - \mathbf{\Phi}\mathbf{a} + \mathbf{y}_1\|_2^2}_{\text{structural constraint}} + \underbrace{\lambda\|\mathbf{z}\|_1 + \frac{\rho_2}{2}\|\mathbf{z} - \mathbf{W}\boldsymbol{\mu} + \mathbf{y}_2\|_2^2}_{\text{sparse constraint}}$$

Solution: Alternating direction method of multipliers (ADMM)

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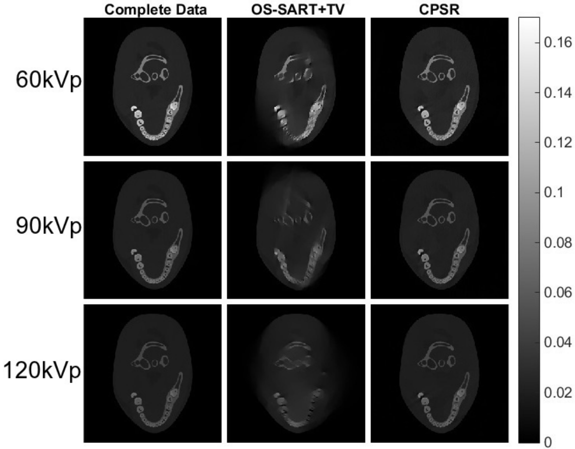
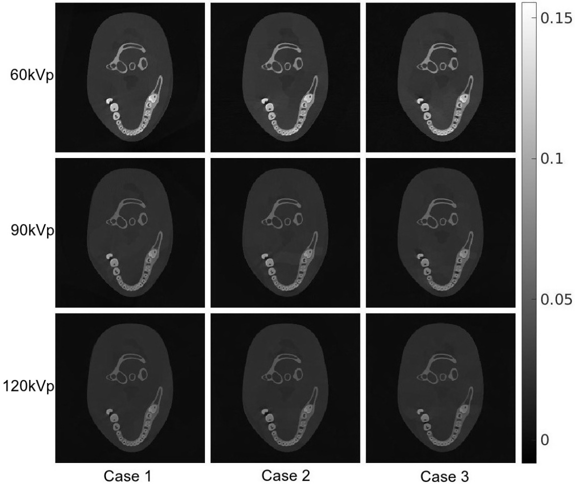
- Scan
  - modality: Fan beam
  - X-ray energies: 120kVp, 90kVp, 60kVp
  - Projection data for each energy: 75 (views) x 320 (detectors)
- Reconstruction
  - Image size: 256×256
  - Clustering number: $k = 100$
  - Reconstruction algorithm: OS-SART
  - Sparse constraint: Total variation in wavelet space



- Complete Data: Independent reconstruction from 180° plus fan beam angle projection data
- OS-SART: Independent reconstruction using OS-SART + TV constraint from 75° angular coverage projection data.
- View 3: CPSR method from 75° angular coverage projection data.

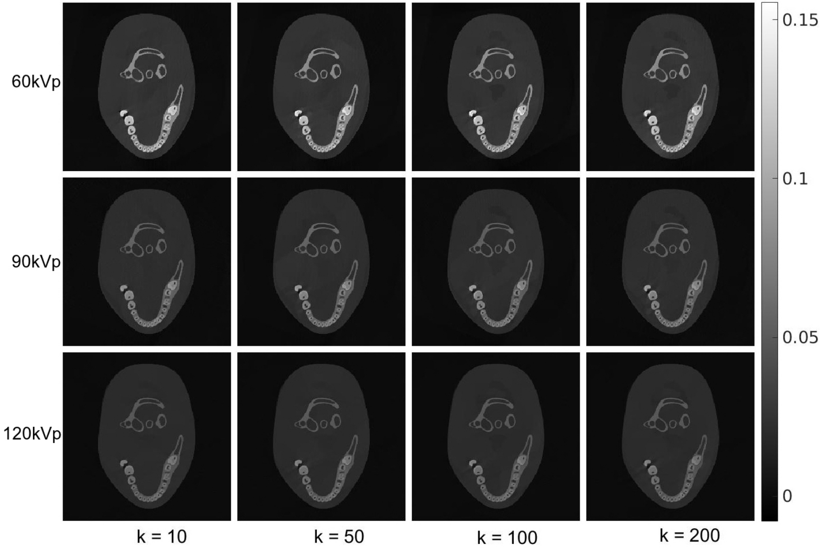


Compare results from different view configurations:

Case 1: $V_{120kVp} = [0°, 75°]$
$V_{90kVp} = [120°, 195°]$
$V_{60kVp} = [240°, 315°]$

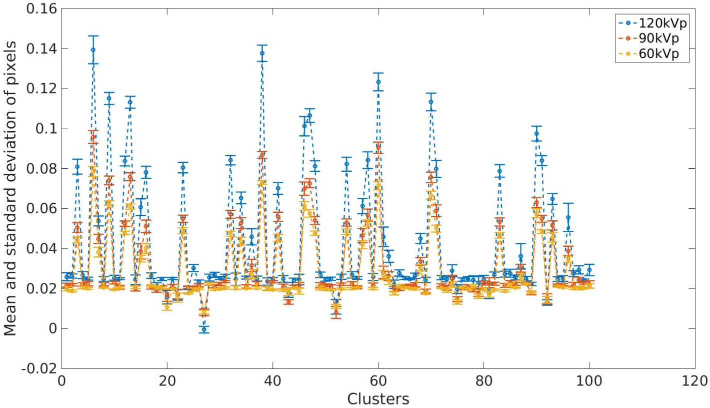
Case 2: $V_{120kVp} = [30°, 105°]$
$V_{90kVp} = [150°, 225°]$
$V_{60kVp} = [270°, 345°]$

Case 3: $V_{120kVp} = [60°, 135°]$
$V_{90kVp} = [180°, 255°]$
$V_{60kVp} = [300°, 375°]$



60kVp
90kVp
120kVp
k = 10
k = 50
k = 100
k = 200
0.15
0.1
0.05
0

- Assumption of identical pixel value within one cluster may be too strong.
- Our method is flexible and tolerates some variation within each cluster by assigning a weight on the prior structural constraint term.



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- Multi-energy CT scan and reconstruction [Shen and Xing 2015]
- Compressed sensing
  1. sparsity[Sidky, Kao, and Pan 2006; Sidky and Pan 2008]
  2. low rank[Gao et al. 2011]
- Limited-angle CT[Jin et al. 2012]
- Sparse dictionary learning[Cao and Xing 2013]

- Largely reduce the projection data required.

From 180° plus fan beam angle to at least 75°.

- Design and implement a reconstruction approach using joint clustering prior and sparsity in wavelet space.
- Solve the limited angle problem by leveraging the coherence among all data at different energies.
- Our method enable flexible angular configuration and broaden spectral CT system design.

Thank you!

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Main References

Cao, Meng and Yuxiang Xing (2013). "Limited angle reconstruction with two dictionaries". In: *Nuclear Science Symposium and Medical Imaging Conference (NSS/MIC), 2013 IEEE*. IEEE, pp. 1–4.

Gao, Hao et al. (2011). "Multi-energy CT based on a prior rank, intensity and sparsity model (PRISM)". In: *Inverse problems* 27.11, p. 115012.

Jin, Xin et al. (2012). "Anisotropic total variation minimization method for limited-angle CT reconstruction". In: *SPIE Optical Engineering+ Applications*. International Society for Optics and Photonics, pp. 85061C–85061C.

Shen, Le and Yuxiang Xing (2015). "Multienergy CT acquisition and reconstruction with a stepped tube potential scan". In: *Medical physics* 42.1, pp. 282–296.

Sidky, Emil Y, Chien-Min Kao, and Xiaochuan Pan (2006). "Accurate image reconstruction from few-views and limited-angle data in divergent-beam CT". In: *Journal of X-ray Science and Technology* 14.2, pp. 119–139.

Sidky, Emil Y and Xiaochuan Pan (2008). "Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization". In: *Physics in Medicine and Biology* 53.17, p. 4777. URL: http://stacks.iop.org/0031-9155/53/i=17/a=021.