Limited-angle Multi-energy CT using Joint Clustering Prior and Sparsity Regularization

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Outline

- Introduction
- Methodology
- Experiments
- Summary
• **Spectral (Multi-energy CT)**
  ▶ differentiates materials
  ▶ wide applications: extracting veins and kidney stones, detecting chemical elements (iodine, barium)

• **Problems**
  ▶ The data required for reconstruction is multiplied
  ▶ longer scan time, more cost, more dose

• **Goal**
  ▶ Design an easy-to-implement scanning strategy
  ▶ Lower does, cost and acquisition/reconstruction time (less angular views)
  ▶ mitigate limited-angle artifacts
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Methodology
Proposed data acquisition strategy: Multi-arc scan

(a) Scanning trajectory
(b) Voltage switching

Requirement: The angular coverage of all X-ray beams is no less than 180° plus fan beam angle.
Methodology
Main Difficulty: Severe limited-angle Problem

- Limited-angle problem:
  - violates data sufficient condition: 180° plus fan beam angle coverage
  - severe artifacts
  - hard to eliminate the artifacts using compressed sensing

- Solutions:
  - Independent reconstruction will unavoidably encounter limited-angle artifacts.
  - Leverage the structural coherence of images at all energies
  - jointly reconstruct images at all energy channels
Methodology
Pre-reconstruction

Combine the projection data from all energies to pre-reconstruct a prior image.

\[ \mu_p = \arg \min_{\mu} \| H \mu - p \|_2^2 \]

where

\[ H = \begin{pmatrix} H_1 & \cdots & H_{NE} \\ H_2 & \vdots & \ddots \\ \vdots & & \ddots & H_{NE} \end{pmatrix} \quad p = \begin{pmatrix} p_1/\|p_1\|_1 \\ p_2/\|p_2\|_1 \\ \vdots \\ p_{NE}/\|p_{NE}\|_1 \end{pmatrix} \]

Each independent \( H_i \) is ill posed, but the combined \( H \) not.

Figure: Prior image

L. Shen and Y. Xing, Medical physics, 42(1): 282, 2015
Solution:

Data fidelity + Regularization
Solution:

Data fidelity + Regularization

\[ \text{Prior} + \text{Sparsity} \]
Methodology
Clustering

- Assumption:
  1. The number of tissues within the object is limited;
  2. Each tissue is spatially continuous;
  3. The pixels within one tissue share an identical value.

- Clustering:
  1. k-means clustering on the prior image;
  2. choosing features \((x, y, \mu(x, y))\). \((x, y)\): coordinates, \(\mu(x, y)\): pixel values;
  3. The image is divided into \(k\) patches;
  4. These \(k\) patches keep some structural details.

Figure: Clustering
Construct a constraint from the clustering

- \( k \) patches: \( \Omega_c = \{(x, y) \mid \text{labeled with } c\}, c = 1, 2, \ldots, k. \)
- construct a dictionary

\[
\mu = \Phi a = \sum_{c=1}^{k} a_c \varphi_c
\]

where
- \( \Phi = (\varphi_1, \varphi_2, \ldots, \varphi_k) \in \mathcal{R}^{N \times k} \) dictionary matrix
- \( \varphi_i \) basis vector (element)
- \( \varphi_{ij} = 1_{i \in \Omega_j} = \begin{cases} 1 & i \in \Omega_j \\ 0 & i \notin \Omega_j \end{cases} \)
Joint Clustering Prior and Sparsity Regularization (CPSR) model

\[
\arg\min_{\mu} \frac{1}{2} \| H\mu - p \|_2^2 + \lambda \| W\mu \|_1 \quad \text{s.t.} \quad \mu = \Phi a
\]

(2)

Incorporates the structural constraint into general compressed sensing frame.

- \( \| H\mu - p \|_2^2 \) linear projection model
- \( \| W\mu \|_1 \) sparse constraint. \( W \) denotes wavelet transform.
- \( \mu = \Phi a \) structural constraint

Augmented Lagrangian Function:

\[
L(\mu, a, z, y_1, y_2) = \frac{1}{2} \| H\mu - p \|_2^2 + \frac{\rho_1}{2} \| \mu - \Phi a + y_1 \|_2^2 + \lambda \| z \|_1 + \frac{\rho_2}{2} \| z - W\mu + y_2 \|_2^2
\]

Solution: Alternating direction method of multipliers (ADMM)
• Introduction
• Methodology
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• Summary
Numerical Experiments

Settings

- **Scan**
  - modality: Fan beam
  - X-ray energies: 120kVp, 90kVp, 60kVp
  - Projection data for each energy: 75 (views) x 320 (detectors)

- **Reconstruction**
  - Image size: 256 x 256
  - Clustering number: \( k = 100 \)
  - Reconstruction algorithm: OS-SART
  - Sparse constraint: Total variation in wavelet space
Numerical experiments
Reconstruction results

- Complete Data: Independent reconstruction from 180° plus fan beam angle projection data.
- OS-SART: Independent reconstruction using OS-SART + TV constraint from 75° angular coverage projection data.
- View 3: CPSR method from 75° angular coverage projection data.
Numerical experiments
Impact of view selection on reconstruction

Compare results from different view configurations:

Case 1: \( V_{120kV_p} = [0^\circ, 75^\circ] \)
\( V_{90kV_p} = [120^\circ, 195^\circ] \)
\( V_{60kV_p} = [240^\circ, 315^\circ] \)

Case 2: \( V_{120kV_p} = [30^\circ, 105^\circ] \)
\( V_{90kV_p} = [150^\circ, 225^\circ] \)
\( V_{60kV_p} = [270^\circ, 345^\circ] \)

Case 3: \( V_{120kV_p} = [60^\circ, 135^\circ] \)
\( V_{90kV_p} = [180^\circ, 255^\circ] \)
\( V_{60kV_p} = [300^\circ, 375^\circ] \)
Numerical experiments
Impact of the clustering number on reconstruction

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Numerical experiments
Impact of the clustering constraint on reconstruction

- Assumption of identical pixel value within one cluster may be too strong.
- Our method is flexible and tolerates some variation within each cluster by assigning a weight on the prior structural constraint term.
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• Multi-energy CT scan and reconstruction [Shen and Xing 2015]
• Compressed sensing
  1. sparsity[Sidky, Kao, and Pan 2006; Sidky and Pan 2008]
  2. low rank[Gao et al. 2011]
• Limited-angle CT[Jin et al. 2012]
• Sparse dictionary learning[Cao and Xing 2013]
• Largely reduce the projection data required. From 180° plus fan beam angle to at least 75°.

• Design and implement a reconstruction approach using joint clustering prior and sparsity in wavelet space.

• Solve the limited angle problem by leveraging the coherence among all data at different energies.

• Our method enable flexible angular configuration and broaden spectral CT system design.
Thank you!

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